

1- **Tick right (✓) or false (x) as appropriate (6 marks)**

a) The initial value of $x(k) = \begin{cases} a^k, & k \geq 2 \\ 0, & \text{otherwise} \end{cases}$ is $x(0) = 1$ () (3 marks)

b) $Z^{-1} \left| \frac{0.5}{(2+z^{-1})} \right| = (-0.5)^k$ () (3 marks)

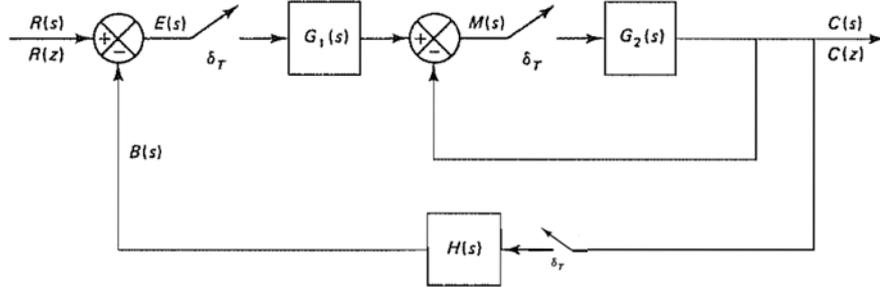
2- **Find the initial and final values of X(z) where (7 marks)**

$$X(z) = \frac{0.5z^2 - z}{(1 - z^{-1})(3z^2 - 4z + 1)}$$

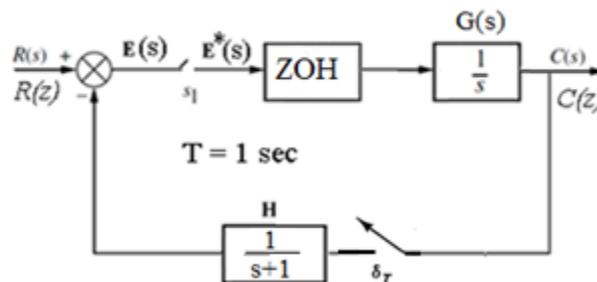
3- **Use the Convolution Integral Method to find the inverse z transform of (7 marks)**

; Confirm your results using another method. $X(s) = \frac{1 - e^{-sT}}{s^2(2s+1)}$

4- **Obtain the pulse transfer function of the system below: (8 marks)**



5- **For the discrete data system shown below: (12 marks)**



- Find the discrete time transfer function (6marks)
- Use the Jury test to determine the stability of the system when it is subjected to a unit step input; i.e. $R(s) = \text{unit step input}$. (6 marks)

See underneath for some given useful information

Table of Laplace and Z-transforms

	$X(s)$	$x(t)$	$x(kT)$ or $x(k)$	$X(z)$
1.	-	-	Kronecker delta $\delta_0(k)$ 1 $k = 0$ 0 $k \neq 0$	1
2.	-	-	$\delta_0(n-k)$ 1 $n = k$ 0 $n \neq k$	z^{-k}
3.	$\frac{1}{s}$	$1(t)$	$1(k)$	$\frac{1}{1-z^{-1}}$
4.	$\frac{1}{s+a}$	e^{-at}	e^{-akT}	$\frac{1}{1-e^{-aT}z^{-1}}$
5.	$\frac{1}{s^2}$	t	kT	$\frac{Tz^{-1}}{(1-z^{-1})^2}$
6.	$\frac{1}{(s+a)^2}$	te^{-at}	kTe^{-akT}	$\frac{Tze^{-aT}z^{-1}}{(1-e^{-aT}z^{-1})^2}$
7.	$\frac{s}{(s+a)^2}$	$(1-at)e^{-at}$	$(1-akT)e^{-akT}$	$\frac{1-(1+aT)e^{-aT}z^{-1}}{(1-e^{-aT}z^{-1})^2}$
8.	-	-	a^k	$\frac{1}{1-az^{-1}}$
9.	-	-	a^{k-1} $k = 1, 2, 3, \dots$	$\frac{z^{-1}}{1-az^{-1}}$

The z transform is given as:

$$X(z) = \sum_{k=0}^{\infty} x(kT)z^{-k}$$

Initial value: $x(0) = \lim_{z \rightarrow \infty} X(z)$

Final value theorem: $x(\infty) = \lim_{z \rightarrow 1} [(1-z^{-1})X(z)]$

The convolution integral - simple poles

$$K_j = \lim_{s \rightarrow s_j} \left[(s - s_j) \frac{X(s)z}{z - e^{Ts}} \right]$$

The convolution integral - repeated poles

$$K_i = \frac{1}{(n_i - 1)!} \lim_{s \rightarrow s_i} \frac{d^{n_i-1}}{ds^{n_i-1}} \left[(s - s_i) \frac{X(s)z}{z - e^{Ts}} \right]$$

$$\text{ZOH} = \frac{1 - e^{-Ts}}{s}$$